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**BTECH**  
**(SEM I) THEORY EXAMINATION 2023-24**  
**ENGINEERING MATHEMATICS-I**

TIME: 3HRS

M.MARKS: 70

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

## SECTION A

1. Attempt all questions in brief.

2 x 7 = 14

Q no.	Question	Marks	C O
a.	Find the product and sum of the eigen values for $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ .	2	1
b.	Find all symmetry in the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$ .	2	2
c.	Calculate the error in $R$ if $E = RI$ and possible errors in $E$ and $I$ are 30% and 20% respectively.	2	3
d.	Determine the value of $\Gamma \frac{1}{4} \Gamma \frac{3}{4}$ .	2	4
e.	Prove that $B(p, q) = B(p + 1, q) + B(p, q + 1)$	2	4
f.	Prove that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.	2	5
g.	Find a unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$ .	2	5

## SECTION B

2. Attempt any three of the following:

7 x 3 = 21

a.	Solve the system of homogenous equations: $x_1 + x_2 + x_3 + x_4 = 0$ , $x_1 + 3x_2 + 2x_3 + 4x_4 = 0$ , $2x_1 + x_3 - x_4 = 0$	7	1
b.	If $u = y^2 e^{y/x} + x^2 \tan^{-1} \left( \frac{x}{y} \right)$ , show that (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$ .	7	2
c.	Expand $f(x, y) = e^x \cos y$ about the point $\left(1, \frac{\pi}{4}\right)$ by Taylor's series.	7	3
d.	Evaluate the integral $\iint_D (y - x) dx dy$ ; by changing the variables, D: Region in xy-plane bounded by the lines $y - x = -3$ , $y - x = 1$ , $y + \frac{1}{3}x = \frac{7}{3}$ , $y + \frac{1}{3}x = 5$ .	7	4
e.	Find the directional derivative of $f(x, y, z) = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of the tangent to the curve $x = a \sin \theta$ , $y = a \cos \theta$ , $z = a\theta$ at $\theta = \pi/4$ .	7	5

## SECTION C

3. Attempt any one part of the following:

7 x 1 = 7

a.	Determine eigen vectors for the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ .	7	1
b.	Determine $A^{-1}$ , $A^{-2}$ and $A^{-3}$ if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem.	7	1



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<b>4. Attempt any one part of the following:</b>		<b>7 x 1 = 7</b>	
a.	If $y = \cos(m \sin^{-1} x)$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ . Also find $(y_n)_0$ .	7	2
b.	If $z = f(x, y)$ , $x = e^u + e^{-v}$ , $y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$	7	2
<b>5. Attempt any one part of the following:</b>		<b>7 x 1 = 7</b>	
a.	If $u^3 + v^3 = x + y$ , $u^2 + v^2 = x^3 + y^3$ , then show $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2 - x^2}{2uv(u-v)}$ .	7	3
b.	The pressure P at any point $(x, y, z)$ in space is $P = 400xyz^2$ . Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method.	7	3
<b>6. Attempt any one part of the following:</b>		<b>7 x 1 = 7</b>	
a.	Find the volume of the solid bounded by the coordinate planes and the surface $\left(\frac{x}{a}\right)^{1/2} + \left(\frac{y}{b}\right)^{1/2} + \left(\frac{z}{c}\right)^{1/2} = 1$ .	7	4
b.	Prove that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$ .	7	4
<b>7. Attempt any one part of the following:</b>		<b>7 x 1 = 7</b>	
a.	Applying Gauss Divergence theorem, evaluate $\iint_S [e^x dydz - ye^x dzdx + 3zdx dy]$ , where S is the surface of the cylinder $x^2 + y^2 = c^2$ , $0 \leq z \leq h$ .	7	5
b.	Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ , where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and hence show that $\nabla^2 \left(\frac{1}{r}\right) = 0$ .	7	5